Limiting Profit Shifting in a Model with Heterogeneous Firm Productivity*

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Abstract

This paper analyzes measures that limit firms' profit shifting activities in a model that incorporates heterogeneous firm productivity and monopolistic competition. Such measures, e.g. thin capitalization rules, have become increasingly widespread as governments have reacted to growing profit shifting activities of multinational companies. However, besides limiting profit shifting, such rules entail costs. As the regulations can only focus on the means to shift profits, not on profit shifting itself, they impose costs on all firms, no matter whether these firms shift profits abroad or not. In the model, these costs force some firms to exit the market. Thus, as the resulting lower competition makes the remaining firms more profitable, regulations to limit profit shifting may even increase the aggregate amount of profits shifted abroad. From a welfare point of view, it can be optimal not to limit profit shifting at all.

Keywords: profit shifting, heterogeneous firms, tax competition

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1 Introduction

With growing financial integration, multinational companies have increasingly shifted profits abroad to reduce their tax payments. Profit shifting is an effective method to lower tax payments: Egger et al. (2010) find that multinationals pay over 50% less taxes than similar domestic firms in high tax countries.[1] Governments have reacted. With the help of targeted changes to the tax code they have tried to secure their respective tax bases. Examples for such measures are stricter transfer pricing rules or thin capitalization rules.

Such regulations have become widespread during the last years. Between the mid-nineties and 2005, for example, the share of EU-27 countries that imposes thin capitalization rules has doubled, from about 30% to 60% (see Buettner et al., 2012). Thin capitalization rules restrict the deduction of interest payments for tax purposes to a certain percentage of earnings. In Germany, for example, interest payments (net of interest expenses) can only be deducted if their value is less than 30% of earnings before interest, taxes, depreciation and amortization (EBITDA). Such rules are not restricted to borrowing from affiliates, but comprise all kinds of debt finance. Due to non-discrimination rules (for example by the EU), they apply to most corporations, even if they are not active internationally. The benefit of such a broad thin capitalization rule is that it effectively limits profit shifting via debt finance.[2]

However, such broadly applicable regulations also have disadvantages. They are badly targeted, as they also apply in cases that have nothing to do with profit shifting.[3] In extreme cases, it is even possible that tax payments accrue under such rules even if the firm makes a loss (see Homburg, 2007).[4] Further costs arise as

[1] Hines and Rice (1994), Huizinga and Laeven (2008) and Weichenrieder (2009), among others, provide further empirical evidence on profit shifting.

[2] There are several empirical analyses of thin capitalization rules, which confirm that such rules indeed have a significant effect on firms' decisions. Buettner et al. (2012) find that thin capitalization rules decrease the use of internal debt, but result in higher external debt. Blouin et al. (2014) show that such rules reduce a firm's aggregate interest expenses. Weichenrieder and Windischbauer (2008) and Overesch and Wamser (2010) reach similar conclusions.

[3] For example, the German thin capitalization rule applies also if a group is only active nationally, and debt mostly stems from bank financing.

[4] Homburg (2007) gives the following example of a corporation making a loss of 20 million euros,

firms hire consultants or choose inefficient strategies to comply with the regulations. For example, under a thin capitalization rule, firms may abstain from internal debt financing even when it would otherwise be optimal (e.g. for investments by affiliates who face high interest rates).

In this paper, I set up a model that incorporates such costs along with the (beneficial) limitation of profit shifting. I then use this model to analyze the effects of limiting profit shifting on welfare and on the aggregate sum of profits shifted abroad. Firms in the model are heterogeneous in their productivity and compete under monopolistic competition. Including heterogeneous productivity is crucial to this analysis of limiting profit shifting, as it allows to model that the effects of this specific tax policy differ among firms with different productivity levels.

The key result of the model is that strengthening a limitation on profit shifting does not necessarily lead to less profit being shifted abroad on aggregate. As the costs of such regulations force some firms out of the market, there is less competition, so that the remaining firms become more profitable. It is therefore possible that the absolute amount of profits shifted abroad increases, even though only a smaller percentage of profits can be transferred. Furthermore, additional firms may start to shift profits abroad.

Regulations to limit profit shifting have further effects, besides the ambiguous effect on the amount of profits shifted itself. As such rules force some firms to exit the market, consumers have fewer varieties from which to choose, which implies a welfare loss. The overall welfare effect depends on the market situation: If firms have high market power, it is best if governments do not limit profit shifting possibilities. If firm productivity is very heterogeneously distributed, profit shifting should be limited, as relatively many firms engage in profit shifting activities to begin with.

Limiting profit shifting is also more likely to be favorable if the costs of profit shifting are relatively low. As such costs have fallen during the last decades due to increasing global integration, this result is in line with the empirical evidence of increased regulation against profit shifting presented above.

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with net interest expenses of 60 million euros and 10 million euros of depreciation allowances. The German thin capitalization rule limits deductible interest expenses to 15 million euros, resulting in a taxable profit of 25 million euros (implying a tax payment of about 7.5 million euros despite making a loss).

This paper is part of the literature that combines models of heterogeneous firm productivity, which are commonly used in international trade theory since Melitz (2003), with the analysis of tax policy. A first part of this literature studies competition for internationally mobile firms (Baldwin and Okubo, 2009; Davies and Eckel, 2010; Haufler and Stähler, 2013). Baldwin and Okubo (2009a) and Bauer et al. (2014) analyse tax-cut-cum-base-broadening tax reforms, and Pflüger and Suedekum (2013) study entry subsidies in the context of firm heterogeneity. Becker (2013) and Bauer and Langenmayr (2013) consider the interaction between taxes and foreign direct investment. Closest to this paper, Krautheim and Schmidt-Eisenlohr (2011) study profit shifting in a model with monopolistic competition among heterogeneous firms. In their model, the most productive firms shift *all* of their profits to a tax haven. In contrast, this paper analyzes the case when the government has a second instrument at its disposal, namely regulations that limit profit shifting. Due to such regulations, firms can only partially shift profits abroad.

A different line of literature examines specific policy measures to limit profit shifting. Hong and Smart (2010) consider if the presence of tax havens is desirable from the perspective of high-tax countries. In an extension they consider the case that the high-tax country imposes thin capitalization rules. Haufler and Runkel (2012) focus explicitly on thin capitalization rules, but in contrast to this paper, the firms' internationalization decision is not endogenous in their model. Instead, they assume that only some firms are active internationally, and firms are otherwise identical.

This paper is structured as follows. The next section introduces the reader to the model and derives a first result on the aggregate amount of profits shifted abroad. Section 3 analyzes the optimization problems of the two countries in more detail. Some numerical simulations in section 4 clarify the theoretical results. Section 5 concludes.

2 Model

The model consists of two countries, the "home market" and the "tax haven". The tax haven is small; all production takes place in the home market. The econ-

omy of the home market comprises two sectors. One of them is a numeraire sector that produces a homogeneous good with a single factor (labor) under perfect competition using a technology with constant returns to scale. The final good in this sector is freely traded, its price is normalized to unity. In the second sector, firms with heterogeneous productivity manufacture differentiated goods under monopolistic competition.[5] The cost of production consists of constant, firm-specific marginal costs a_i and fixed production costs c. Marginal costs of all potentially active firms follow a Pareto distribution in the interval $[0, 1]$.[6] The cumulative distribution function of the marginal cost is given by

$$G(a) = a^{\gamma}\ , \qquad \gamma > 1. \tag{1}$$

The Pareto distribution implies that higher values of a_i are more likely than lower values, i.e. that relatively few very productive firms exist. Firms are more heterogeneous when the shape parameter γ is lower.[7]

Firms in the differentiated goods sector compete under Dixit-Stiglitz monopolistic competition: Each firm offers a product that is, from the consumers' point of view, only imperfectly substitutable by other goods. Therefore, firms have some market power. Consumers' preferences are given by

$$U = \mu \ln X_I + \beta X_G + X_N, \quad \text{with} \quad X_I = \left(\int_{i \in \Theta} x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 > \mu > 0. \tag{2}$$

X_I and X_N represent the quantities consumed of the differentiated and numeraire goods, respectively. X_G is a public good, financed by tax revenue, and enters the utility function weighted by a factor $\beta > 1$. μ shows the importance of the differentiated goods relative to the numeraire good. Θ is the set of all differentiated goods. The Dixit-Stiglitz parameter σ can be interpreted as the (constant) elasticity of substitution.

[5] To focus on profit shifting, it is assumed that there is no trade in the differentiated goods sector. This is common in the literature, see e.g. Krautheim and Schmidt-Eisenlohr (2011).

[6] The Pareto distribution is a good approximation of the empirically observed distribution of firm sizes (see Axtell, 2001). Its use is common in the literature, see e.g. Helpman et al. (2004) or Krautheim and Schmidt-Eisenlohr (2011).

[7] With a lower value of the parameter γ, more low-cost firms exist. If $\gamma \to 1$, marginal cost is uniformly distributed, which represents the highest degree of heterogeneity possible in this model. In the opposite case of $\gamma \to \infty$, firms are homogeneous (with marginal costs equal to a_0).

Maximizing the utility function (2) subject to the budget constraint, the demand for a particular variety of the differentiated good is given by

$$x_i = \frac{\mu}{\int p_i^{-(\sigma-1)}} \, p_i^{-\sigma} = \frac{\mu}{p_i^{\sigma}} P^{\sigma-1}, \qquad \text{with} \quad P = \left(\int_{i \in \Theta} p_i^{-(\sigma-1)} di \right)^{-\frac{1}{\sigma-1}}, \tag{3}$$

where p_i is the price of variety i, and P is the CES price index. Aggregate demand for X_I is thus given by $X_I = \frac{\mu}{P}$.

Firms facing this demand function maximize their profits. Therefore they set their prices as a constant mark-up over marginal cost,

$$p\left(a_i\right) = \frac{\sigma}{\sigma - 1} a_i. \tag{4}$$

The price is higher if the elasticity of substitution is lower, i.e. if firms have more monopoly power.

Thus, pre-tax profits of firms are given by the following equation, with the second equality taking optimal price and quantity decisions into account[8]

$$\begin{aligned} \pi_i - c &= \left(p_i - a_i\right) \cdot x_i - c \\ &= \frac{\mu}{\sigma} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} \left(\frac{P}{a} \right)^{\sigma-1} - c. \end{aligned} \tag{5}$$

The most efficient firms (i. e. the firms with the lowest marginal cost a_i) are the most profitable.

Profits are taxed at a constant marginal rate t_H in the home country. However, firms have the possibility to shift profits to the tax haven, where the profit tax rate t_X is lower than in the home country ($0 < t_X < t_H < 1$).[9] To shift profits, firms have to incur a fixed cost, f. As the cost of profit shifting is fixed, only the most profitable firms choose to shift their profits abroad (see Krautheim and Schmidt-Eisenlohr, 2011).

In reality, many regulations limit profit shifting. Such regulations have to target the means of profit shifting, i.e. transfer pricing or internal financing structures. Due

[8] In line with the previous literature, I assume that $\gamma - \sigma + 1 > 0$ to ensure that profits are finite.

[9] As the tax haven has no firms of its own, it collects tax revenue only if it sets a lower tax rate than the home market. Otherwise, no firm would be willing to shift profits.

to limited information, the government cannot fully differentiate between legitimate intra-firm transactions and profit shifting. It thus has to use broader rules that also hinder activities other than profit shifting.[10] An example are thin-capitalization rules, which dictate that only interest expenses of up to a certain fraction of profits can be subtracted from earnings for tax purposes for all firms. On the one hand, this limits the possibilities for profit shifting via debt. On the other hand, it increases *all* firms' financing costs, as they have to comply with these rules or face higher tax rates, also if they did not intend to shift profits abroad.

In the model, the government of the home country can impose such regulations, which limit the maximum percentage of profits α that firms are able to shift abroad. This, however, imposes costs on all firms. These costs can be interpreted in manifold ways: Firms may choose inefficient strategies to comply with the regulations, or have to hire costly consultants. If thin capitalization rules are used, financing costs may rise. As many regulations (such as the German thin capitalization rule, see fn. 3) apply to all firms, such costs arise even if the firm does not engage in profit shifting activities. Thus, limits on profit shifting impose some costs on firms, as they have to comply with the regulations, e.g. find different sources of financing if internal debt is limited by a thin capitalization rule. To represent all these different costs, every firm in the model bears an additional cost of $(1-\alpha)\tau$. The parameter τ scales the severity of the burden, which also depends on the strictness of the limitation on profit shifting, $(1-\alpha)$.[11]

Due to the fixed costs of production and profit shifting limitations, not all potential firms are productive enough to be in business. A zero-profit condition determines

[10] In EU countries, non-discrimination laws even prohibit regulations that affect only internationally active firms.

[11] While this cost is the same across firms in *absolute* value, it is highest for low-productivity firms in *relative* terms. This is realistic, as unproductive firms are hit harder for example by thin capitalization rules (as they have fewer self-financing possibilities and are more dependent on debt finance). Nevertheless, all firms have some additional costs due to such regulations (such as consultants and investment distortions). Empirical evidence confirms that small, low-productivity firms have higher tax compliance costs relative to their profits than large, highly productive firms: Slemrod and Blumenthal (1996) presents survey evidence that the tax compliance cost *per employee* is about $ 1,200 for firms with less than thousand employees, but only about $ 60 for firms with over forty thousand employees.

the cut-off value a_τ, that is, the cost coefficient of the least productive firm in the market:

$$[p(a_\tau)\, x(a_\tau) - a_\tau x(a_\tau) - c](1 - t_H) - (1 - \alpha)\tau = 0. \tag{6}$$

Solving this condition for a_τ yields

$$a_\tau = \frac{\sigma - 1}{\sigma}\left(\frac{\mu}{\sigma}\frac{1 - t_H}{c(1 - t_H) + (1 - \alpha)\tau}\right)^{\frac{1}{\sigma - 1}} P. \tag{7}$$

If the tax rate in the home country or the cost of the limitation on profit shifting is higher, fewer firms are in the market. More firms are active in larger markets (as measured by μ). For $\alpha = 1$ equation (7) collapses to the case without a limitation on profit shifting.

Every firm can incur a fixed cost f to shift some of its profits abroad.[12] As the most profitable firms have the most to gain from avoiding taxes, and given that the cost of profit shifting is fixed, only firms with marginal costs below a level a_P shift profits abroad. This cut-off is determined by the following indifference condition, which already takes into account that fixed costs c and the burden of the limitation on profit shifting $(1 - \alpha)\tau$ have to be borne in both cases:[13]

$$(1 - t_H)\,\pi(a_P) = (1 - t_H)(1 - \alpha)\,\pi(a_P) + (1 - t_X)\,\alpha\pi(a_P) - f. \tag{8}$$

The left hand side of equation (8) represents the case in which the firm pays taxes only in the home country. On the right hand side, it shifts profits into the tax haven. As the cost of profit shifting, f, is fixed, the firm always shifts as much of its profit abroad as is possible. It is assumed that the cost of profit shifting is not deductible from the firm's taxable base.[14]

[12] I assume that fixed costs are such that not all firms engage in profit shifting, so that the least productive firm in the market (i.e. the firm with marginal costs of a_τ) is not avoiding taxes.

[13] The tax-deductible fixed cost of production, c, is always deducted in the home country. Due to the higher tax rate there, this is optimal for the firm.

[14] The assumption of no deductibility is justified if the costs of profit shifting lie in distortions or soft costs (such as language barriers or an inability to effectively monitor employees) that arise because an investment (e.g. a sales and distribution office) is undertaken in a tax haven instead of in a high-tax country. It is also common in the literature with heterogeneous firms, see e.g. Krautheim and Schmidt-Eisenlohr (2011). An alternative assumption would be that these costs are deductible in the tax haven, which would not change the analysis qualitatively.

Inserting equation (5) for the profits, the marginal cost level a_P is given by

$$a_P = \frac{\sigma - 1}{\sigma}\left(\frac{\mu}{\sigma}\frac{\alpha\left(t_H - t_X\right)}{f + \alpha c\left(t_H - t_X\right)}\right)^{\frac{1}{\sigma-1}} P. \tag{9}$$

Firms with marginal costs under a_P shift as much of their profits as possible abroad; the other firms (with marginal costs $a_i \in [a_P,\ a_\tau]$) prefer to pay taxes on all profits in the home country, as the costs of profit shifting are – for them – too high relative to their earnings. A higher tax rate in the home country or a lower tax rate abroad induce more firms to shift profits abroad. Note that a_P depends only indirectly (via the price index) on the costs that profit shifting limitations impose on all firms.

The cut-off values a_τ and a_P depend on the price index. Using the definition of the price index and combining it with equations (4) and (7), the equilibrium value of the price index is

$$P = \left(\int_0^{a_\tau} p_i^{-(\sigma-1)} di\right)^{-\frac{1}{\sigma-1}} = \frac{\sigma}{\sigma-1}\left(\frac{\gamma - \sigma + 1}{\gamma}\right)^{\frac{1}{\gamma}}\left(\frac{1 - t_H}{c\left(1 - t_H\right) + \left(1 - \alpha\right)\tau}\right)^{-\frac{\gamma-\sigma+1}{\gamma(\sigma-1)}}. \tag{10}$$

Lastly, let us consider optimal quantities of the numeraire good X_N and the public good X_G. Demand for the numeraire good is given by $X_N = I - \mu$, whereby income I consists of labor income L and profit income. The public good is financed by tax revenue T, $X_G = T$. Thus, optimal quantities of the numeraire and the public good depend on aggregate profits and their taxation.

To look at this in more detail, consider the tax base in both countries (i.e. aggregate profits without deducting the burden imposed by the limitation on profit shifting).These are, for the home country (Π) and tax haven (Π^*) respectively:

$$\begin{aligned}\Pi &= \int_{a_P}^{a_\tau}\left(\pi_i - c\right) dG\left(a\right) + \int_0^{a_P}\left[\left(1 - \alpha\right)\pi_i - c\right] dG\left(a\right) \qquad (11)\\ &= \frac{\mu}{\sigma}\left[1 - \alpha\left[\frac{\left[c\left(1 - t_H\right) + \left(1 - \alpha\right)\tau\right]\alpha\left(t_H - t_X\right)}{\left(1 - t_H\right)\left[f + \alpha\left(t_H - t_X\right)c\right]}\right]^{\frac{\gamma-\sigma+1}{\sigma-1}} - c\frac{\left(\gamma - \sigma + 1\right)\left(1 - t_H\right)}{\gamma\left[c\left(1 - t_H\right) + \left(1 - \alpha\right)\tau\right]}\right],\end{aligned}$$

$$\Pi^* = \int_0^{a_P} \alpha\pi_i dG\left(a\right) = \alpha\frac{\mu}{\sigma}\left[\frac{c\left(1 - t_H\right) + \left(1 - \alpha\right)\tau}{1 - t_H}\frac{\alpha\left(t_H - t_X\right)}{f + \alpha\left(t_H - t_X\right)c}\right]^{\frac{\gamma-\sigma+1}{\sigma-1}}. \tag{12}$$

The tax base in the home country, equation (11), can be interpreted as the sum of all profits (i. e. the tax base without any profit shifting, $\frac{\mu}{\sigma}$) less the profits shifted to the tax haven (the second term) and aggregate fixed costs.

What determines how much profit is shifted abroad on aggregate? The tax base in the tax haven rises if the tax difference between the two countries increases. It falls if firms have higher costs to shift profits (higher f) or if the demand for differentiated goods in the home market is lower (lower μ). If profit shifting limitations impose a greater burden, the tax base is lower, as there are fewer firms in the market. The tax base in the haven is also smaller if the firms are more heterogeneous (lower γ).

Importantly, it is not always the case that a limitation on profit shifting leads indeed to less profit being shifted abroad on aggregate. Differentiating equation (12) with respect to α, it becomes clear that counteracting effects are at work:

$$\frac{\partial \Pi^*}{\partial \alpha} = \int_0^{a_P} \left(\pi_i + \alpha \frac{\partial \pi_i}{\partial \alpha} \right) dG\left(a\right) + \frac{\partial a_P}{\partial \alpha} \pi\left(a_P\right). \tag{13}$$

The first term reflects the effects on the intensive margin, that is, the change in the amount of profit each firm shifts abroad. First, there is a direct effect $\left(\int_0^{a_P} \pi_i dG\left(a\right)\right)$, as a change in α changes the percentage of profits that each firm is allowed to shift abroad. Secondly, there is an indirect effect: By strengthening a limitation on profit shifting (lowering α), the government crowds some firms out of the market. For the remaining firms, the market becomes less competitive, thus rendering them more profitable. Thus, possibly, if the increase in profitability is strong enough, these firms shift *more* profits abroad despite the profit shifting regulation. The second term captures an effect on the extensive margin, that is, on the number of firms that shift profits. As all active firms become more profitable, it is possible that firms that did not shift profits abroad before start to do so after it is strengthened.[15] The following proposition summarizes these effects.

Proposition 1 (Effectiveness of Limits on Profit Shifting)**.** *Stricter limitations on profit shifting do not necessarily lead to less profit shifted abroad on aggregate. Such regulations are only effective if the burden associated with them is relatively small.*

Proof. By inspection of equation (13) and using equations (5), (9) and (10) it follows that $\frac{\partial \Pi^*}{\partial \alpha} > 0 \Leftrightarrow \tau < \frac{c(1-t_H)(\sigma-1)\left[c\alpha(t_H-t_X)+f+\frac{\gamma-\sigma+1}{\sigma-1}\right]}{[c\alpha(t_H-t_X)+f](\alpha\gamma-\sigma+1)-(1-\alpha)(\gamma-\sigma+1)}$ ■

[15] Again, two counteracting effects are at work on the extensive margin, as can be seen by inspection of equation (9). A stricter regulation makes profit shifting less attractive per se, but the increase in profits (due to less competition, i.e. a higher price index P) may change that.

This result is driven by the market exit of some small firms, which occurs because of the compliance costs associated with regulations limiting profit shifting. To gauge the likelihood that such costs force some firms out of the market, let us consider some real-world numbers: In a survey of U.S. firms, Slemrod and Blumenthal (1996) find average tax compliance costs of $ 1.6 million per firm. Even the smallest firms in their sample report compliance costs of $ 0.7 million. About 40% of these compliance costs arise due to regulations concerning foreign-source income and are thus directly related to measures against profit shifting (Blumenthal and Slemrod, 1995). In addition, limitations on profit shifting cause distortions (such as a different choice of financing sources) that have further costs for firms. In total, it is plausible that measures against profit shifting force some firms out of the market.

A numerical simulation in section 4 will shed some more light on this result. First, however, I derive and discuss the conditions that determine the optimal tax rates and limitation on profit shifting in the following section.

3 Optimal Tax Policies

3.1 Optimization of the Tax Haven

The tax haven sets its tax rate t_X to maximize its tax revenue. As it has no firms of its own, maximizing tax revenue is the optimal policy also from a welfare point of view. Thus, its optimization problem is

$$\max t_X \Pi^*. \tag{14}$$

Solving (14) yields the tax haven's best response function,

$$t_X = t_H - \frac{1}{2}\frac{\sqrt{4t_H cf\alpha(\sigma-1)(\gamma-\sigma+1)+f^2\gamma^2}-f\gamma}{c\alpha(\sigma-1)}. \tag{15}$$

The tax haven reacts to stricter limitations on profit shifting (lower α) by lowering its tax rate. If the tax rate in the home country is increased, the tax haven responds in kind, but raises its tax rate by less than the home country has raised hers. If firms are very heterogeneous (low γ), the tax rate is higher: There are more productive firms, and those firms are the first to shift profits. Therefore, the tax haven can

attract quite a lot of firms even if its tax rate is not that low. If the elasticity of substitution, σ, is lower, firms have more monopoly power and realize higher profits. In this case, the tax haven sets a lower tax rate.

3.2 Optimization of the Home Country

The home country can decide about two policy instruments, the tax rate and the degree to which it restricts profit shifting. The government sets these to maximize social welfare. Welfare is given by the indirect utility function of the representative consumer, which follows from using $X_I = \frac{\mu}{P}$, $X_G = T = t_H \Pi$ and $X_N = I - \mu$ in the utility function (2):

$$V = L - \mu + \mu \ln\left(\frac{\mu}{P}\right) + (1 + (\beta - 1)\, t_H)\, \Pi + (1 - t_X)\, \Pi^* - N^* f - N_\tau\, (1 - \alpha)\, \tau. \quad (16)$$

Note that income I consists of labor income L and (after-tax) profit income. The fixed costs of profit shifting, f, are paid by all $N^* = a_P^\gamma$ firms that shift profits abroad. $N_\tau = a_\tau^\gamma$ marks the total mass of firms in the market. Fixed costs of production, c, are already deducted from aggregate profits.

The first order conditions for the optimal limitation on profit shifting and the optimal tax rate are

$$\frac{\partial V}{\partial \alpha} = \frac{-\mu}{P}\frac{\partial P}{\partial \alpha} + [1 + (\beta - 1)\, t_H]\, \frac{\partial \Pi}{\partial \alpha} + (1 - t_X)\, \frac{\partial \Pi^*}{\partial \alpha} - f\frac{\partial N^*}{\partial \alpha} - \frac{\partial N_\tau}{\partial \alpha}\, (1 - \alpha)\, \tau + N_\tau \tau = 0, \quad (17)$$

$$\frac{\partial V}{\partial t_H} = \frac{-\mu}{P}\frac{\partial P}{\partial t_H} + [1 + (\beta - 1)\, t_H]\, \frac{\partial \Pi}{\partial t_H} + \Pi\, (\beta - 1) + (1 - t_X)\, \frac{\partial \Pi^*}{\partial t_H} - f\frac{\partial N^*}{\partial t_H} - \frac{\partial N_\tau}{\partial t_H}\, (1 - \alpha)\, \tau = 0. \quad (18)$$

Due to the analytical complexity of the model, these first order conditions cannot be solved explicitly for t_H and α. In section 4, numerical solutions will be derived and shown graphically. However, before doing so, I will give some intuition for the various effects a limitation on profit shifting has on welfare.

First, to interpret the effects of such a regulation better, equation (17) can be rewritten as

$$\frac{\partial V}{\partial \alpha} = \frac{-\mu}{P}\frac{\partial P}{\partial \alpha} - [t_H\, (\beta - 1) + t_X]\, \frac{\partial \Pi^*}{\partial \alpha} - f\frac{\partial N^*}{\partial \alpha} - [1 + (\beta - 1)\, t_H]\, c\frac{\partial N_\tau}{\partial \alpha} - \left[(1 - \alpha)\, \frac{\partial N_\tau}{\partial \alpha} - N_\tau\right] \tau \quad (19)$$

using that $\frac{\partial \Pi}{\partial \alpha} = -\frac{\partial \Pi^*}{\partial \alpha} - c\frac{\partial N_\tau}{\partial \alpha}$.

The first term of equation (19) captures the effect that limiting profit shifting has on consumption. This term is always positive, showing that stricter regulations (lower α) have a negative effect on welfare: As some firms exit the market, fewer varieties are available to the consumer.[16]

The main advantage of a regulation that limits profit shifting is supposedly that less profits are shifted abroad. The change in the volume of profits shifted has two effects, which are captured in the second term of equation (19). First, stricter profit shifting rules increase tax revenues in the home country. Second, less income is lost to tax payments in the tax haven (from the home country's point of view, taxes paid on profits in the tax haven are a pure loss, as they neither generate tax revenue nor profit income). Moreover, as shown by the third term of equation (19), if fewer firms shift profits, less profit income is lost due to the fixed cost f, which firms incur to shift profits. Note, however, that it is not clear whether such a rule really leads to less profits being shifted abroad (see proposition 1).

The fourth term of equation (19) reflects that as fewer firms are in the economy, fewer firms incur the fixed costs of production, c. As this cost is tax-deductible, this also has implications for tax revenue. Lastly, the strictness of regulations influences the severity of the burden that is associated with such a limitation on profit shifting. First, lowering α implies that this burden affects fewer firms, as some firms exit the market. However, a stricter limit on profit shifting also implicates that this burden is higher for all firms. This second effect is always stronger than the first, implying that the total effect is negative (i.e. the last term of equation (19) is always positive).

Proposition 2 (Welfare effects of regulations to limit profit shifting)**.** *The welfare effects of strengthening a limitation on profit shifting are ambiguous and given by equation* (19)*. Besides the positive effect of keeping profits in the country, such a regulation has further effects due to the market exit of some firms. This decreases competition and makes consumers worse off as they have fewer varieties from which to choose, but may increase tax revenue (see proposition 1).*

[16] As the CES price index reflects the price of the optimized consumption bundle, it unambiguously falls when fewer varieties are available, even though these varieties were the most expensive in the market.

Next, let us consider the effects of a change of the tax rate in the home country, t_H. Again, it is helpful to rewrite the first order condition (18) as

$$\frac{\partial V}{\partial t_H} = \frac{-\mu}{P}\frac{\partial P}{\partial t_H} - (\beta - 1)\left(\Pi + t_H \frac{\partial \Pi}{\partial t_H}\right) + t_X \frac{\partial \Pi^*}{\partial t_H} - f\frac{\partial N^*}{\partial t_H} - \left[c + (1-\alpha)\,\tau\right]\frac{\partial N_\tau}{\partial t_H} \quad (20)$$

The first term again captures the effect on consumption: If the tax rate is higher, it is more difficult to be profitable enough to stay in the market despite the excess burden of regulations to limit profit shifting. Thus, a higher tax rate implies fewer varieties in the market, thereby decreasing welfare. The second term captures the effect of a tax rate increase on tax revenues. First, there is a direct effect: A higher tax rate implies higher revenues, everything else being equal. However, there is also a negative indirect effect, as the tax base decreases because the higher tax rate leads to more profit shifting. The additional profit shifting also implies that income is "lost" in the tax haven, because more profits are taxed there. This effect is represented by the third summand of equation (20). As more firms incur the fixed costs of profit shifting, income is further reduces, as the fourth term shows. Lastly, there also is a positive effect of market exit due to higher tax rates: As fewer firms are active, the dead weight loss of the limitation on profit shifting affects fewer firms and fewer firms have to pay the fixed costs of production.

These various effects allow no clear conclusion whether limiting profit shifting is desirable, given that it imposes costs on all firms. To see the effects of such a limitation more clearly, the next section looks at some numerical simulations of the modeled economy.

4 Numerical Analysis

4.1 Simulation of Proposition 1

The theoretical model has shown that it is not clear that a regulation that limits profit shifting always succeeds in its aim of decreasing the amount of profits that is shifted to a tax haven (see proposition 1). In the following, numerical simulations will illustrate this. Their results are shown in figure 1.

The graphs clarify how the tax policy of the home country affects the aggregate amount of profits shifted abroad. It compares the aggregate value of profits shifted

abroad in the case with a limitation on profit shifting (dark plane) and without such a limitation (light plane). The optimal response of the tax haven (i.e. the optimal t_X) is taken into account.

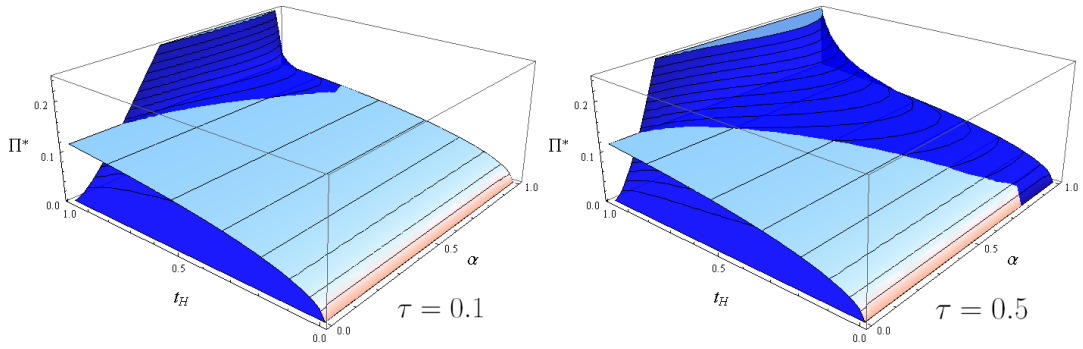

Figure 1: *Aggregate profits shifted abroad, with a limitation imposed (dark plane) and without such a limitation (light plane). Parameter values:* $L = 1, \mu = 0.5, \beta = 3, f = c = 0.1, \gamma = 1.5, \sigma = 2$. *On the left,* $\tau = 0.1$, *on the right,* $\tau = 0.5$.

On the two axes with the independent variables are t_H and α, which constitute the home country's tax policy. The graph in the benchmark case without a limitation on profit shifting (i.e. the light layer) is independent of α, which is drawn on the right-hand axis. Aggregate profits in the tax haven go to zero if either $\alpha \to 0$ or $t_H \to 0$, as then there is either no possibility or no incentive to shift profits abroad. Note, however, that this does not allow any inference about welfare.

The graphs clarify that only for some combinations of α and t_H a rule which limits profit shifting actually leads to less profits being shifted abroad on aggregate. Even with relatively low costs of profit shifting regulation (left graph), a strict regulation may induce more profit shifting if the tax rate is high. If the burden is relatively high (right graph), very strict regulations to limit profit shifting may be counterproductive no matter what the tax rate is. As explained above, this happens because such regulation decreases competition in the market.

4.2 Simulation of Proposition 2

Lastly, let us consider the welfare effects, which where described in section 3 and summarized in proposition 2. A numerical analysis of the model confirms that it is not always optimal to limit profit shifting if this entail costs for all firms.

However, the simulations also show that if a limitation is welfare-increasing at all, then the government should set is as strict as possible, i.e. set $\alpha \to 0$ (see appendix A).[17] Thus, the optimum in the model economy is always a corner solution, setting α either to 0 or to 1.

It depends on the characteristics of the economy (i.e. on parameters) whether a country chooses to prohibit profit shifting or not. The following two figures (figures 2 and 3) show how market and firm characteristics influence whether profit shifting should be barred.

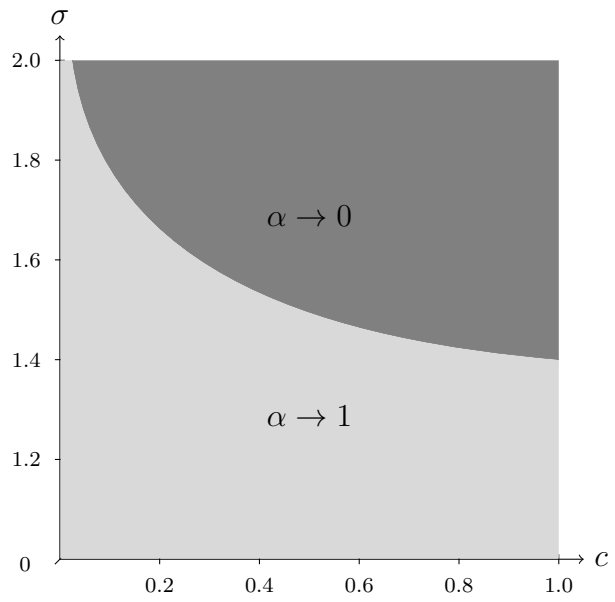

Figure 2: *Prohibiting profit shifting for different market characteristics. Parameter values:* $\gamma = 1.5, L = 1, \mu = 0.5, \beta = 3$ *and* $f = \tau = 0.1$.

Figure 2 summarizes the results of simulations comparing welfare without a limitation on profit shifting (i.e. $\alpha \to 1$) and after its introduction for different market characteristics. These are on the one hand the elasticity of substitution, σ, which is also a measure for competition in the market, and fixed costs, c, which represent barriers to entry into the market. The darker area represents parameter

[17] A prohibition of all profit shifting possibilities is not what we observe in reality. Note, however, that in the model it is actually feasible to deter all profit shifting, which is hardly the case in reality. It should hence be interpreted as the government limiting profit shifting as much as it can, while in the other alternative the government chooses not to limit profit shifting at all.

constellations under which it is favorable to prohibit profit shifting.

Profit shifting should not be limited if there are relatively many firms in a relatively uncompetitive market. If the elasticity of substitution is low, then it is more important for consumers to have as many firms in the market as possible. Hence, the utility loss of loosing additional varieties is higher. In contrast to what might be the first intuition, this effect is stronger when many firms are in the market (low fixed costs c), because the additional fixed costs of limiting profit shifting become more important when other fixed costs are low.

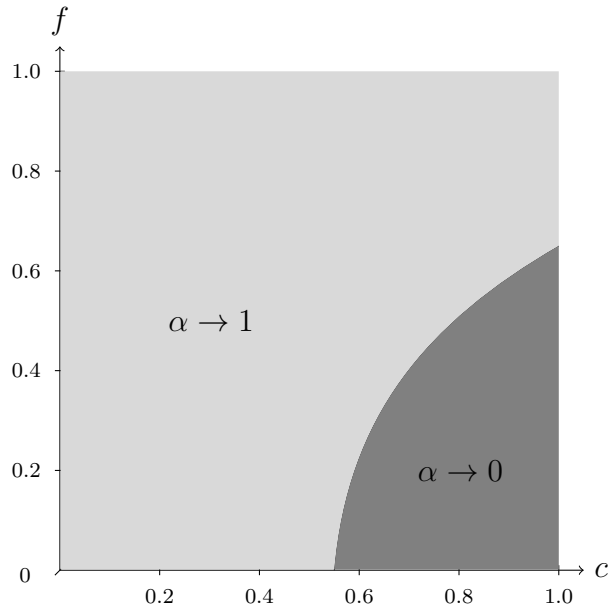

Figure 3: *Prohibiting profit shifting for different firm characteristics. Parameter values:* $\gamma = 1.5, L = 1, \mu = 0.5, \beta = 3, \tau = 0.1$ *and* $\sigma = 1.5$.

A further interesting aspect is the interplay of the different firm characteristics, namely fixed costs c and the costs of profit shifting, f, which is depicted in figure 3. It is intuitive that the benefit from limiting profit shifting is smaller if few firms shift profits due to high costs f, especially because the burden imposed by regulation to hinder this falls on all firms. However, high fixed costs c make it more likely that profit shifting should be limited. If fixed costs are high, the market consists mainly of highly profitable firms, which are more likely to shift profits abroad, thus increasing the benefit of limiting profit shifting.

The degree of firm heterogeneity also influences whether profit shifting should

be prohibited or not. Heterogeneity is measured by γ. High heterogeneity (a low γ) implies that the distribution of firm productivity approaches a uniform distribution, i.e. there are many firms with very high or low productivity levels.[18] If firms are very heterogeneous, limiting profit shifting is more favorable. This is the case because with high heterogeneity, there are relatively many large, productive firms, which would shift all of their profits abroad otherwise, and relatively few small, unproductive firms, which are affected negatively (or even forced out of the market) by the regulations.

5 Conclusion

This article has analyzed the various effects and welfare implications of limiting profit shifting. It points out that regulations that aim to limit profit shifting may curb competition by forcing some firms out of the market. By rendering the remaining firms more profitable, it is possible that *more* profits are shifted abroad on aggregate after the introduction of a regulation that is supposed to prohibit or limit profit shifting.

In the introduction it was mentioned that such measures, e.g. thin capitalization rules, have increasingly been introduced or strengthened during the last years. The model also offers explanations for this by clarifying the effect of different parameters on the likelihood that limiting profit shifting increases welfare. It becomes clear that lower costs of profit shifting, which may have resulted from increasing financial integration, make limiting profit shifting more beneficial.

[18] A low level of firm heterogeneity in this sense would be the case if there are many firms with similar (low) productivity levels and only very few highly productive firms.

A Appendix: Simulation Results

The following table 1 states some results of numerical simulations of the model. The fixed parameters are $\tau = 0.1, \mu = 0.5, \beta = 3$ and $L = 1$.

Parameters																
c	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5
f	0.1	0.1	0.5	0.5	0.1	0.1	0.5	0.5	0.1	0.1	0.5	0.5	0.1	0.1	0.5	0.5
γ	1.5	1.5	1.5	1.5	2	2	2	2	1.5	1.5	1.5	1.5	2	2	2	2
σ	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	2	2	2	2	2	2	2	2
t_H	0.99	0.20	0.99	0.87	0.97	0.26	0.96	0.85	0.79	0.88	0.94	0.89	0.80	0.85	0.89	0.86
t_X	0.43	0.09	0.36	0.37	0.35	0.10	0.26	0.29	0.53	0.60	0.64	0.59	0.46	0.45	0.46	0.43
α	1	1	1	1	1	1	1	1	0	0	1	0	1	0	1	0

Table 1: Results of the numerical simulation.

It becomes clear that is always either optimal not to limit profit shifting at all ($\alpha \to 1$) or to prohibit profit shifting completely ($\alpha \to 0$). This is also clarified by the following graphs (figure 4), which plot welfare depending on the tax rate t_H and on α.

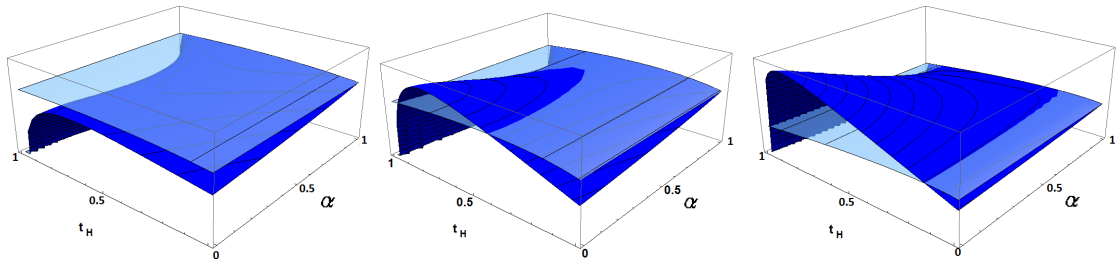

Figure 4: *3D-Plots of simulated welfare levels, varying σ (from left to right: $\sigma = 1.75, \sigma = 2$ and $\sigma = 2.25$). Parameter values: $L = 1, \mu = 0.5, \beta = 3, f = c = \tau = 0.1$ and $\gamma = 1.5$.*

The graphs clarify that it is not always welfare-increasing to introduce a limitation on profit shifting: In the graph on the left, welfare with such a limitation is always lower than in the benchmark case where profit shifting is not limited. If such a rule should be introduced, it is optimal to set it as strict as possible (that is, at the left side of the graph).

Welfare is depicted for different values of the elasticity of substitution, σ, in figure 4. It shows that limiting profit shifting becomes more favorable if the elasticity of substitution is higher. In that case, it is less important for the consumer to have different varieties available. Hence, the negative effect of a limitation on profit

shifting (i.e. market exit by some firms and thus the loss of these varieties) is less pronounced if σ is high.

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